Nonuniform sediment transport in alluvial rivers

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• Introduction
• New development on the hiding and exposure factor
• Threshold for incipient motion of nonuniform sediment
• Fractional transport rate of nonuniform bed-load
• Fractional transport rate of nonuniform suspended load
• Procedure for computing fractional transport rate of bed-material load
• Testing of the proposed transport formulas
• Conclusions
Determining the critical condition for sediment incipient motion and the sediment transport rate is very important in hydraulic engineering.

After DuBoys published his research on the bed-load transport rate in 1879 and Shields (1936) proposed the curve for the prediction of the critical bed shear stress of incipient motion, the uniform sediment movement has been extensively investigated and the transport mechanism is reasonably well understood.

The state-of-art for estimating the nonuniform sediment transport is still inadequate.

It is needed to consider the hiding and exposure effect in the modeling of nonuniform sediment transport.
The correction factors were related to bed-material size

\[ \eta_i = f \left( \frac{d_i}{d_m}; \text{ or } \frac{d_i}{d_{50}} \right) \]  \hspace{1cm} (1)

\( d_i \): the diameter of the \( i \)th fraction of sediment;
\( d_m \) and \( d_{50} \): the arithmetic mean and 50% sieve diameters of bed materials.
Misri et al. (1984)

1) Assumed
   the motion of fine particles $\rightarrow$ the lift force
   the motion of coarse particles $\rightarrow$ the drag force

2) Proposed a semi-theoretical hiding-exposure correction factor

This correction factor was revised subsequently by Samaga et al. (1986a), Mittal et al. (1990) and Patel and Range Raju (1996).

\[ \eta_i = f_2 \left( M, \frac{\tau'_b}{\tau_c}, \frac{\tau_{*i}}{\tau_c} \right) \]  \hspace{1cm} (2)

\( M \): the Kramer's uniformity coefficient
\( \tau'_b \): the grain shear stress
\( \tau_c \): the critical shear stress for the arithmetic mean size \( d_m \)
\( \tau_{*i} = \frac{\tau'_b}{[(\gamma_s - \gamma)d_i]} \)
\( \gamma_s \) and \( \gamma \): the specific weights of sediment and fluid, respectively.
The drag and lift forces acting on a particle staying on the bed depend on how it is situated and surrounded by others.

Assume:
- sediment particles are spheres with various diameters

Define:
- the exposure height $\Delta_i$ for a particle with size $d_i$ as the elevation difference between the apexes of this particle and its upstream particle.
If $\Delta_i > 0$, exposed state
$\Delta_i < 0$, hidden state
Assume to follow a uniform probability distribution $f$.
If the upstream particle is $d_i$, $f$ can be expressed as

$$f = \begin{cases} 
1/(d_i + d_j), & -d_j \leq \Delta_i \leq d_i \\
0, & \text{otherwise}
\end{cases} \quad (3)$$

The probabilities of particles $d_i$ hidden and exposed by particles $d_i$ can be obtained from Eq. (3) as

$$p_{hi, j} = p_{bj} \frac{d_j}{d_i + d_j} \quad (4)$$

$$p_{ei, j} = p_{bj} \frac{d_j}{d_i + d_j} \quad (5)$$
The total hidden and exposed probabilities of particles $d_i$

$$p_{hi} = \sum_{j=1}^{N} p_{bj} \frac{d_j}{d_i + d_j}$$  \hspace{1cm} (6)$$

$$p_{ei} = \sum_{j=1}^{N} p_{bj} \frac{d_i}{d_i + d_j}$$  \hspace{1cm} (7)$$

The hiding and exposure factor is defined as

$$\eta_i = \left( \frac{p_{ei}}{p_{hi}} \right)^m$$  \hspace{1cm} (8)$$

$N$: the total number of particle size fractions of nonuniform sediment mixtures

$m$: empirical parameter
Threshold for incipient motion of nonuniform sediment

The formula for determining the critical bed shear stress for the incipient motion of nonuniform sediment

\[
\frac{\tau_{ci}}{(\gamma_s - \gamma)d_i} = \theta_c \left(\frac{p_{ci}}{p_{hi}}\right)^m
\]

(\(\theta_c = 0.03\) and \(m = -0.6\))

Parker et al. (1982) suggested the following threshold for the incipient motion of nonuniform sediment

\[
W_{ri}^* = \frac{q_{bi}(\rho_s/\rho - 1)g}{p_{bi}u_*^3} = 0.002
\]

\(W_{ri}^*\): a reference transport parameter

\(q_{bi}\): the volumetric transport rate per unit width for the \(i\)th fraction of bed-load

\(p_{bi}\): the gradation of the \(i\)th fraction of bed material

\(u_*\): the bed shear velocity
Fig. 2. Measured and calculated critical shear stresses: (a) Newly proposed eq.(9); (b) Egiazaroff's formula eq.(11); (c) Hayashi et al's formula eq.(12).

\[ \theta_{ci} / \theta_c = \begin{cases} \left[ \log \frac{8}{\log (19d_i/d_m)} \right]^2 d_i / d_m \geq 1 \\ d_m / d_i \quad d_i / d_m < 1 \end{cases} \]  

(12)
This type of formulas for the uniform bed-load transport rate or the total transport rate of nonuniform bed-load can be written as

\[
\phi_b = f_3\left(\frac{\tau_b}{\tau_c} - 1\right)
\]  

(13)

- \(\phi_b\): a non-dimensional bed-load transport rate,
- \(q_b / \sqrt{\gamma_s / \gamma - 1} gd^3\)
- \(q_b\): the volumetric bed-load transport rate per unit width
- \(\tau_b\): the total bed shear stress or the bed shear stress due to grain roughness
The bed shear stress can be calculated with

$$\tau_b = \gamma R_b J$$  \hspace{1cm} (15) 

The grain shear stress is defined as

$$\tau_b' = \gamma R_b' J$$  \hspace{1cm} (16)

- $R_b$ : the hydraulic radius of channel bed
- $J$ : the energy slope
- $R_b'$ : the hydraulic radius corresponding to the grain roughness on the bed
- $n$ : the Manning's roughness coefficient for channel bed
- $n'$ : the Manning's roughness corresponding to grain roughness
Laboratory and field data of bed-load

Four sets of laboratory data for nonuniform bed-load
Some of the field data from five natural rivers
The flow and sediment parameters must be measured at the same time.
(flow discharge, velocity, depth, surface slope, bed-load transport rate,
bed-load gradation and bed-material gradation)

Table 1. Flow and sediment parameters of bed-load data.

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Discharge (m³/s)</th>
<th>Velocity (m/s)</th>
<th>Depth (m)</th>
<th>Energy Slope (10⁻³)</th>
<th>d_i (mm)</th>
<th>q_p (10⁻³ m²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samaga (1986a)</td>
<td>0.006-0.015</td>
<td>0.49-0.78</td>
<td>0.06-0.11</td>
<td>4.49-6.93</td>
<td>0.073-2.366</td>
<td>0.04-0.22</td>
</tr>
<tr>
<td>Kuhnle (1993)</td>
<td>0.01-0.03</td>
<td>0.28-0.81</td>
<td>0.101-0.107</td>
<td>0.47-2.22</td>
<td>0.2-10</td>
<td>0.0000015-0.064</td>
</tr>
<tr>
<td>Wilcock (1993)</td>
<td>0.017-0.057</td>
<td>0.26-1.08</td>
<td>0.088-0.12</td>
<td>0.59-16.2</td>
<td>0.21-64</td>
<td>0.00000087-0.22</td>
</tr>
<tr>
<td>Liu (1986)</td>
<td>0.0035-0.023</td>
<td>0.14-0.67</td>
<td>0.03-0.083</td>
<td>1.5-4</td>
<td>0.31-30</td>
<td>0.000049-0.00064</td>
</tr>
<tr>
<td>Susitna River</td>
<td>799-2800</td>
<td>1.8-2.1</td>
<td>2.4-4.4</td>
<td>1.4-2.4</td>
<td>0.062-128</td>
<td>0.028-0.11</td>
</tr>
<tr>
<td>Chulima River</td>
<td>261-348</td>
<td>1.5-1.8</td>
<td>1.7-1.9</td>
<td>0.64-0.74</td>
<td>0.062-128</td>
<td>0.11-0.23</td>
</tr>
<tr>
<td>Black River</td>
<td>20-256</td>
<td>0.44-1.0</td>
<td>0.55-1.9</td>
<td>0.11-0.29</td>
<td>0.062-16</td>
<td>0.0048-0.016</td>
</tr>
<tr>
<td>Tottle River</td>
<td>9.3-248</td>
<td>1.3-2.8</td>
<td>0.39-1.5</td>
<td>1.9-5.5</td>
<td>0.062-32</td>
<td>0.11-0.95</td>
</tr>
<tr>
<td>Yampa River</td>
<td>26.3-447</td>
<td>0.59-1.3</td>
<td>0.65-3.9</td>
<td>0.40-0.87</td>
<td>0.062-32</td>
<td>0.003-0.054</td>
</tr>
</tbody>
</table>
Fig. 3. Relationship for Fractional Transport Rate of Nonuniform Bed-Load.
The suspended load transport rate
the rate of energy available to the alluvial system $\rightarrow \tau U$ ($\tau = \gamma RJ$)
the gravity $\rightarrow$ settling velocity $\omega$ and the critical shear stress $\tau_c$

the independent parameter: $\tau U/\tau_c \omega$

$\tau / \tau_c \rightarrow (\tau - \tau_c) / \tau_c$

The fractional transport rate of nonuniform suspended load has relationship

$$\phi_{si} = f_4 \left[ \left( \frac{\tau}{\tau_{ci}} - 1 \right) \frac{U}{\omega_i} \right]$$

(19)

$$\phi_{si} = q_{si} / \left[ p_{bi} \sqrt{(\gamma_s / \gamma - 1)} gd_i^3 \right]$$
The laboratory data of nonuniform suspended load (Samaga et al. 1986b) and two sets of field data (the Yampa River and the Yellow River) were used to analyze the relationship in Eq. (19).

Fig. 4. Relationship for Fractional Transport Rate of Nonuniform Suspended Load.
The fractional transport rate of nonuniform bed-material load = the fractional transport rates of bed-load Eq.(18) + suspended load Eq.(20).

The following steps are required for this calculation:

1. Divide the nonuniform sediment mixtures into several fractions with different size ranges and determine $d_i$, $\omega_i$ and $p_{bi}$ for each fraction.
2. Calculate the hidden and exposed probabilities $p_{hi}$ and $p_{ei}$.
3. Determine the critical shear stress $\tau_{ci}$ for each size fraction with Eq. (9).
4. Calculate the shear stress $\tau$ and the bed shear stress $\tau_b$ from the known flow velocity, depth and surface slope. For natural rivers, $\tau_b = \gamma h J$ can be used, and for experimental situations, $\tau_b$ should be obtained by eliminating the bank shear stress.
5. Determine the Manning's roughness coefficient $n$ for channel bed (excluding the influence of banks) and the $n \, '( = d_{50}^{1/6} / 20)$ corresponding to the grain shear stress, and then calculate the grain shear stress $\tau_b'$ with Eq. (17).

6. Calculate the non-dimensional excess shear stress: $T_i = \tau_b' / \tau_{ci} - 1$, and then the fractional transport rate $q_{bi}$ for nonuniform bed-load with Eq. (18).

7. Calculate the parameter $(\tau / \tau_{ci} - 1)U / \omega_i$, and then the fractional transport rate $q_{si}$ for nonuniform suspended load with Eq. (20).

8. Sum $q_{bi}$ and $q_{si}$ to obtain the fractional transport rate for nonuniform bed-material load.
Eqs. (18) and (20) are jointly tested against 1859 sets of uniform bed-material load data selected from Brownlie's (1981) data collection by limiting the standard deviation of bed material $\sigma < 1.2$ and the Shields parameter $\theta > 0.055$.

Similar tests using the same data set are also conducted on three widely used bed-material load transport formulas developed by Engelund and Hanson (1967), Ackers and White (1973), and Yang (1973, 1984).

Table 3. Comparison of calculated versus measured transport rates of uniform bed-material load.

<table>
<thead>
<tr>
<th>Error Ranges</th>
<th>Percentages (%) of Calculated Transport Rates in Error Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ackers &amp; White</td>
</tr>
<tr>
<td>$0.8 \leq r \leq 1.25$</td>
<td>37.3</td>
</tr>
<tr>
<td>$0.667 \leq r \leq 1.5$</td>
<td>57.9</td>
</tr>
<tr>
<td>$0.5 \leq r \leq 2$</td>
<td>82.4</td>
</tr>
</tbody>
</table>

Note: $r$ = calculation / measurement.
Eq. (18) is separately tested against 1345 sets of uniform bed-load data selected from the previous 1859 sets of uniform bed-material load data by limiting the Rouse number \( \omega/Ku_* > 2.5 \).

A comparison is also conducted with four widely adopted bed-load transport formulas of Meyer-Peter and Mueller (1948), Bagnold (1966), Engelund and Fredsøe (1976) and van Rijn (1984).

Table 4. Comparison of calculated versus measured transport rates of uniform bed-load

<table>
<thead>
<tr>
<th>Error Ranges</th>
<th>Van Rijn</th>
<th>Engelund &amp; Fredsøe</th>
<th>Bagnold</th>
<th>Meyer-Peter &amp; Mueller</th>
<th>Wu et al</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8 ≤ r ≤ 1.25</td>
<td>14.8</td>
<td>21.4</td>
<td>21.4</td>
<td>21.3</td>
<td>38.7</td>
</tr>
<tr>
<td>0.667 ≤ r ≤ 1.5</td>
<td>25.3</td>
<td>37.4</td>
<td>38.9</td>
<td>39.4</td>
<td>59.3</td>
</tr>
<tr>
<td>0.5 ≤ r ≤ 2</td>
<td>44.0</td>
<td>54.1</td>
<td>57.2</td>
<td>66.2</td>
<td>80.1</td>
</tr>
</tbody>
</table>
Eqs. (18) and (20) are also jointly tested against the nonuniform sediment data collected by Toffaleti (1968).

The tests are also conducted on the Proffit and Sutherland's (1983) modification of Ackers and White's formula, the modified Zhang's formula (Wu and Li, 1992) as well as Karim's formula (1998).

Table 5. Comparison of calculated versus measured fractional transport rates of nonuniform bed-material load

<table>
<thead>
<tr>
<th>Data Source and Number</th>
<th>Percentages (%) of Calculated Fractional Transport Rates in Error Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5 \leq r \leq 2</td>
</tr>
<tr>
<td></td>
<td>PS  Zh  Ka  Wu</td>
</tr>
<tr>
<td>Flume, 196</td>
<td>10.7 56.6 36.7 61.2</td>
</tr>
<tr>
<td>Field, 343</td>
<td>2.6   43.2 46.1 56.0</td>
</tr>
<tr>
<td>Total, 539</td>
<td>5.6   48.1 42.7 57.9</td>
</tr>
</tbody>
</table>

Note: PS=Proffit and Sutherland; Zh=modified Zhang; Ka=Karim; Wu=Newly proposed Eqs. (18)&(20)
The hiding and exposure effect among the particles of nonuniform bed material is proven important in the prediction of nonuniform sediment transport.

The hiding and exposure correction factor developed can account for not only the influence of sediment particle size but also that of bed-material gradation.

The comparisons using the laboratory and field data show that the methodology developed has more advantages than Egiazaroff's and Hayashi et al's approaches which only consider the influence of sediment particle size.

These newly proposed formulas provide better predictions than several existing methods.
Thanks for your attention